LIMITS [1.8] & CONTINUITY [1.7]

NON-TECHNICAL DEFINITION OF A LIMIT (MATH 124):

We define the limit of the function $f(x)$ as x approaches c, written $\lim_{x\to c} f(x)$, to be a number L (if one exists) such that $f(x)$ is as close to L as we want whenever x is sufficiently close to c (but $x \neq c$). If L exists, we write $\lim_{x\to c} f(x) = L$.

Example 1: Explain why $\lim_{x\to 0}\left(\frac{1}{x^2}\right)$ does not exist.

As x approaches zero, $\frac{1}{x^2}$ becomes arbitrarily large, so it cannot approach any finite number L. Therefore we say $\frac{1}{x^2}$ has no limit as $x \to 0$ and we write: $\lim_{x\to 0}\left(\frac{1}{x^2}\right)$ DNE where $DNE \equiv Does\ Not\ Exist.$

If, however, $\lim_{x\to c} f(x)$ does not exist because $f(x)$ gets arbitrarily large on both sides of c, we also say $\lim_{x\to c} f(x) = \infty$.

Since $\frac{1}{x^2} \to \infty$ as $x \to 0^+$ and $\frac{1}{x^2} \to \infty$ as $x \to 0^-$, we also write $\lim_{x\to 0}\left(\frac{1}{x^2}\right) = \infty.$

DEFINITION OF CONTINUITY

The function f is continuous at $x = c$ if the following principles hold:

(1) f is defined at $x = c$, that is $(c, f(c))$ is a point on the graph of f.

(2) [a] $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x)$

[b] $\lim_{x\to c} f(x) = f(c)$

Example 2: Let $g(x) = \begin{cases} (x+1)^2 & if\ x \leq 1 \\ x & if\ x > 1 \end{cases}$ Is $g(x)$ continuous at $x = 1$?

Example 3: Let $h(z) = \frac{5z^2+2}{z^2+1}$ Is $h(z)$ continuous at $z = 3$?